If we wish our iteration procedure to converge to the second mode, it is clear that we must eliminate any contribution due to the first mode. This is accomplished by choosing the matrix **B** so as to force the coefficient of a_1 to zero. Thus, we set

$$\mathbf{B} = [1/M_1(\omega_1)^2]\boldsymbol{\varphi}^{(1)} \tag{9}$$

and the $\boldsymbol{\varphi}^{(1)}$-contribution is removed from the iteration process. The modified equation to be iterated is

$$[\mathbf{D} - [1/M_1(\omega_1)^2]\boldsymbol{\varphi}^{(1)}(\boldsymbol{\varphi}^{(1)})^T\mathbf{M}]\mathbf{A} = (1/\omega^2)\mathbf{A} \tag{10}$$

The result of solving this equation will be second mode and frequency.

The extension of Turner's method to higher modes and frequencies is straightforward. If we have determined k modes and frequencies and wish to find the $(k+1)$-st mode, we simply iterate the following equation:

$$\left\{\left[D - \left(\sum_{j=1}^{k} \frac{1}{M_j(\omega_j)^2}\boldsymbol{\varphi}^{(j)}(\boldsymbol{\varphi}^{(j)})^T\mathbf{M}\right]\right\}\mathbf{A} = 1/\omega^2\,\mathbf{A} \tag{11}$$

Conclusions

Turner's method of matrix iteration for higher modes and frequencies has been developed. It is straightforward to apply and requires no matrix inversions for its use. It is particularly adaptable to use with digital computers and requires only the subtraction of a matrix from the dynamic matrix **D** after each successive mode and frequency are found. This method has been used with great success at Georgia Tech.

References

[1] Bisplinghoff, R. L., Ashley, H., Halfman, R. L., *Aeroelasticity*, Addison-Wesley, Reading, Mass., 1955, pp. 164–172.

[2] Hurty, W. C. and Rubinstein, M. F., *Dynamics of Structures*, Prentice-Hall, Englewood Cliffs, N.J., 1964, pp. 123–130.

[3] Meirovitch, L., *Analytical Methods in Vibrations*, Macmillan, London, 1967, pp. 91–95.

Angle of Attack Increase of an Airfoil in Decelerating Flow

T. Strand*

Air Vehicle Corporation, San Diego, Calif.

IT has been found experimentally that the maximum lift coefficient of a transport airplane in flight is substantially higher than that measured in a wind tunnel when the flight airplane is undergoing an angle of attack increase while decelerating.[1]

This purpose of this Note is to determine, using inviscid theory, the aerodynamic characteristics of a two-dimensional airfoil whose angle of attack α is increasing at a constant rate $\dot\alpha$, and whose velocity U is decreasing at a constant rate $-\dot U$. Thus, let

$$U = U_0 + \dot U t \tag{1}$$

where U_0 is the velocity at $t = 0$. Without loss of generality it may be assumed that the airfoil is a flat plate. To simplify the expressions, it will be assumed that the axis of rotation of the airfoil is located at the midchord. The vertical distance η to an arbitrary point on the flat plate is then given by the following expression:

$$\eta = -x\alpha = -x(\alpha_0 + \dot\alpha t) \tag{2}$$

where x is the horizontal coordinate, and α_0 is the angle of attack at $t = 0$.

To obtain a solution, using the concept of acceleration potential,[2,3] it will be necessary to determine the vertical acceleration a_y of a fluid particle adjacent to the airfoil surface, i.e.,

$$a_y = \frac{d^2\eta}{dt^2} = \frac{\partial^2\eta}{\partial t^2} + 2U\frac{\partial^2\eta}{\partial x\,\partial t} + \frac{dU}{dt}\frac{\partial\eta}{\partial x} + U^2\frac{\partial^2\eta}{\partial x^2} = -2U\dot\alpha - \dot U(\alpha_0 + \dot\alpha t) \tag{3}$$

Let us now introduce a new coordinate system $\bar x$, $\bar y$, which is rotated with respect to the x, y system by the angle α. In this new coordinate system the flat plate airfoil lies along the abscissa. Because $\alpha \ll 1$ we find $a_y \simeq a_{\bar y}$. Defining a complex acceleration function $W(\bar z)[=\phi + i\psi]$, where $\bar z = \bar x + i\bar y$, we have

$$a_{\bar y} = -\partial\psi/\partial\bar x = -2U\dot\alpha - \dot U(\alpha_0 + \dot\alpha t) \tag{4}$$

Integration with respect to $\bar x$ along $\bar y = 0$ on the airfoil yields

$$\psi = [2U\dot\alpha + \dot U(\alpha_0 + \dot\alpha t)]\bar x + C(t) \tag{5}$$

Here $C(t)$ is an integration constant. The flat plate airfoil of chord c is next mapped conformally onto the unit circle $\zeta = e^{i\theta}$, where $i = (-1)^{1/2}$, by

$$\bar z = (c/4)(\zeta + 1/\zeta) \tag{6}$$

Thus $\bar x = (c/2)\cos\theta$, $\bar y = 0$ for corresponding points on the flat plate and on the circle. Equation (5) therefore becomes

$$\psi = [2U\dot\alpha + \dot U(\alpha_0 + \dot\alpha t)](c/2)\cos\theta + C \tag{7}$$

The required acceleration function W, whose imaginary part will reduce to ψ [Eq. (7)] on $\zeta = e^{i\theta}$ and will die out at infinity, is given by

$$W = iA/\zeta + i2C/(\zeta + 1) \tag{8}$$

where

$$A = [2U\dot\alpha + \dot U(\alpha_0 + \dot\alpha t)]c/2 \tag{9}$$

The real part of Eq. (8) yields

$$\phi = A\sin\theta + C\tan(\theta/2) \tag{10}$$

From the general theory of acceleration potentials, we have the following expression for the lift L of an airfoil in unsteady flow

$$L = 2\rho\int_{-c/2}^{c/2}\phi\,dx \tag{11}$$

It is noted that the acceleration potential ϕ is proportional to the instantaneous chordwise pressure difference between the upper and lower surfaces of the airfoil. This being the case, we may, in order to obtain a picture of what is happening, associate the pressure difference with a fictitious effective airfoil meanline yielding this pressure distribution in a steady flow. Denoting the chordwise vortex distribution of the effective meanline by γ, the lift in steady flow becomes

$$L = \rho U\int_{-c/2}^{c/2}\gamma\,dx \tag{12}$$

Received March 31, 1972.

Index Categories: Subsonic and Transonic Flow; Nonsteady Aerodynamics.

* President.

By comparing Eqs. (11) and (12), we now find

$$\frac{\gamma}{U}=\frac{2\phi}{U^2} \tag{13}$$

Thus, by Eqs. (9, 10, and 13)

$$\frac{\gamma}{U}=2(2k_{\dot\alpha}-k_{\dot U})\sin\theta+\frac{2C}{U^2}\tan\frac{\theta}{2} \tag{14}$$

where by definition

$$k_{\dot\alpha}=\frac{\dot\alpha c}{2U} \tag{15}$$

$$k_{\dot U}=\frac{(-\dot U/U)\alpha c}{2U} \tag{16}$$

The first term on the right-hand side of Eq. (14) is recognized as being the vortex distribution of a parabolic arc meanline which is symmetric about the midchord, and whose ideal angle of attack therefore is zero. The second term on the right-hand side is the vortex distribution of a flat plate airfoil at angle of attack. The constant C determines the lift lag of the additional distribution in unsteady flow. This constant can be found by applying the boundary condition at the airfoil surface, making use of the Cauchy-Riemann condition, and performing an integration. This will not be done here, since it is believed that the lag is insignificant for all cases of practical interest to maximum lift. Note that there is no lift lag for the effective camber change of the airfoil.

The effective camber η_c may be found by substituting Eq. (14) into the following expression from thin airfoil theory, and integrating.

$$\frac{d\eta_c}{d\bar x_0}=(1/2\pi)\int_{-c/2}^{c/2}\frac{(\gamma/U)}{\bar x-\bar x_0}\,d\bar x \tag{17}$$

The result is

$$\eta_c/c=(2k_{\dot\alpha}-k_{\dot U})[(1/4)-(\bar x/c)^2] \tag{18}$$

From thin airfoil theory it is also known that the angle for zero lift α_{ZL} of a parabolic meanline is equal to twice the maximum camber, i.e.,

$$-\alpha_{ZL}=k_{\dot\alpha}-\tfrac{1}{2}k_{\dot U} \tag{19}$$

Unsteady flow of the type investigated here will therefore yield a lift coefficient increase ΔC_L at a given angle of attack in the linear region as follows:

$$\Delta C_L=2\pi(k_{\dot\alpha}-\tfrac{1}{2}k_{\dot U}) \tag{20}$$

At least part of this change might be expected to affect the maximum lift coefficient, indicating an increase due to rate of change of angle of attack and a decrease due to deceleration. However, the reader may easily convince himself that the predicted ΔC_L change due to inviscid flow conditions is negligible compared to the test results obtained, both for jet transports and also for helicopter rotor blades. It is concluded that the measured large increases in maximum lift coefficients in unsteady flow are associated with boundary-layer phenomena, rather than being caused by the inviscid flow.

References

[1] Kier, D. A., "Flight Comparison of Several Techniques for Determining the Minimum Flying Speed for a Large, Subsonic Jet Transport," TN D-5806, June 1970, NASA.

[2] Biot, M. A., "Some Simplified Methods in Airfoil Theory," *Journal of the Aeronautical Sciences*, Vol. 9, March 1942, pp. 186–190.

[3] Wood, J. E., "Application of the Acceleration Potential to Aerodynamic Force Calculation in Oscillating Flow," M. S. thesis, Dept. of Aeronautics June 1970, Naval Postgraduate School, Monterey, Calif.

Evaluation of Reissner's Correction for Finite Span Aerodynamic Effects

B. K. Donaldson*
The Boeing Company, Wichita, Kansas

REFERENCES 1 and 2 set forth Reissner's method for calculating the effect of finite span upon the unsteady airloads acting upon an harmonically oscillating wing. Reissner's method is limited to incompressible flow, but it is applicable to any wing planform, at least down to an aspect ratio of three. An advantage of the Reissner theory is that it is only a modification of basic strip theory,[3] which simplifies its application. In addition, the calculations required by the Reissner method are less extensive than those required by lifting surface theory. The accuracy of the Reissner method is the concern of this Note. To evaluate that accuracy, the Reissner method was applied to ten moderate-aspect-ratio model wings. The Reissner flutter speeds that were the result of those analyses are compared to experimental flutter speeds obtained from repeated wind-tunnel testing, and also to basic strip theory results.

The ten model wings which are the subjects of the analyses were all of sufficiently low aspect ratio that finite span effects were significant. The model wing geometries, mass distributions, and stiffness distributions were typical of actual wing construction. Each semispan model had a planform area of 1620 in.[2] A typical semispan length was 37.65 in. This size permitted good precision in measuring model parameters. With one exception, each model wing represented a single variation in the geometric design parameters of aspect ratio, thickness ratio, taper ratio, and leading-edge sweepback angle. Thus, the wind tunnel data is an indication of the effect of these wing parameters on flutter speed. See Table 1. The high speed wings, for which these were models, were designed for the same airplane. Each wing required a different tail loading, so each wing was designed to have the necessary strength to support a particular wing loading. As a consequence of this and the different configurations, the model wings had different stiffness and mass properties.

Because of their single-spar, weighted-segment-type construction, these models were well suited for a structurally simple evaluation of unsteady airload theories. Each wing spar supported nine lead weighted, balsa wood panels whose width was one-tenth of the semispan length. An additional inboard panel was fixed to the fuselage support from which the wings were cantilevered. Each of the nine outboard panels was fixed on the spar at the panel midpoint, except for the most outboard panel which, for different models, was fixed at varying, more inboard points. There was no elastic interaction between panels other than that provided by the single spar. Thus, an elastic-axis, discrete-mass mathematical description of the model wings was entirely appropriate. The ratios of the spar area moments of inertia were such that just three degrees-of-freedom were indicated for each wing panel: vertical bending deflection, bending slope angle, and torsional twist. The center of gravity locations, first mass moments, moments of inertia, and product of inertia were carefully measured or estimated for each panel. The spar stiffness design data were corrected by experimental deflection

Received February 2, 1972; revision received April 10, 1972. The author would like to thank L. J. Topp for general guidance, and D. Fall, G. Wells, and J. Symes, of the Boeing Co. for assistance with the calculations. The experimental work was supervised by G. Luessen of the Boeing Co. The author would also like to thank H. Ashley, University of Maryland, for discussing this Note.
Index category: Aeroelasticity and Hydroelasticity.

* Structural Dynamics Engineer, Structural Dynamics Unit. Presently Assistant Professor, University of Maryland, College Park, Md. Member AIAA.